UNSCRIPTED MATHS: EMERGENCE AND IMPROVISATION

MIKE ASKEW

Monash University

Mike.Askew@monash.edu

It may seem that improvisational drama and primary mathematics are two diametrically opposed disciplines, the former being based around emergence and uncertainty and the latter based around predictability and certainty. In this paper I argue that creative mathematics teaching and learning requires a certain amount of unpredictability and that, particularly with regard to problem solving, learners' solutions have a certain quality of emergence that is not dissimilar to improvisational drama. By examining children's solutions to non-routine problems I consider what mathematics education might gain from attending to the discipline of improvisation.

Theoretical background

Collaborative emergence

Sawyer (2001) traces the origins of the concept of emergence to work in 1875 by the philosopher George Henry Lewes and Lewes' distinction between two types of effects: resultants and emergents. The main qualities of emergent effects, Sawyer argues, are that outcomes cannot be fully understood or predicted by studying the constituent parts, as illustrated by Lewes' example of the effect of water emerging from the combination of oxygen and hydrogen. Understanding the properties of water cannot fully be achieved by reduction to the study of the properties of oxygen and hydrogen (although quantum physics now overturns this claim). This non-reductionist aspect of emergent phenomena means that they are multiplicative rather then additive in their nature (Davis & Simmt, 2003). Sawyer does not define resultant effects but I take these to be those effects that are predictable through the study of their component parts, typified by the behaviour of billiard balls.

Although the concept of emergence has been developed since Lewes' time, particularly in the physical sciences, it probably began to have most impact on educational research with the development of artificial intelligence systems that displayed intelligent behaviour based on simple, local rules of interaction and without the need for a central leader. Thus models of how insect colonies create complex structures or birds fly in symmetrical flocks became canonical examples of emergent systems (Clark 1997). From these simple forms of emergence it has generally become accepted that group behaviour can be considered as emergent when there is no structured plan for the group to follow, and where there is no leader directing the group (Sawyer, 1999). Classrooms and students are, however, fundamentally different from anthills and ant, or flocks and seagulls, in the range of actions and agency available to the participants. To distinguish between systems where there is interaction but not agency, in the sense that individuals within the system can intentionally change the direction of what is emerging, I am using Sawyer's phrase of *collaborative emergence* to encompass phenomena "that result from the collective activity of social groups" (Sawyer, 1999, p. 449).

Whilst not necessarily using the terminology of collaborative emergence, most teachers and researchers might consider group behaviour as emergent when there is no pre-determined plan or script that a leaderless group is following. In the context of this paper, the groups that I am considering to be engaged in collaborative emergent activity are pairs of children working on finding a solution to non-routine mathematics problems. As the children were not given any direction from the teacher as to how to solve the problems, nor were they assigned particular roles within their pairs (in particular, neither child was asked to act as 'leader' of the pair) their problem solving activity fits with Sawyer's criteria to be classed as emergent.

A key theoretical and analytical shift in treating group activity as collaboratively emergent is that "interaction among constituent components leads to overall system behaviour that could not be predicted from a full and complete analysis of the individual components of the system" (Sawyer, 2000, p. 183).

Performance and improvisation

Performance in some of the educational literature has perjorative overtones. For example, Dweck (2000) talks of 'performance oriented' learners as learners who are keen to be seen to 'performing' in correct and acceptable ways and that as such 'performance' is not always linked to understandings. Similarly, there are overtones sometimes of being taught to 'perform' in the 'training' sense of the word.

In contrast to such views of performance as not creative or allowing for agency, I am using the term in the sense used by Holzman (2000), in that the majority of our activity could be thought of as having an element of performance, and that one reading of Vygotsky is that we learn and develop through performing.

> Performative pyschology is based in an understanding of human life as primarily performative, that is, we collectively create our lives through performing (simultaneously being who we are and who we are becoming) (Holzman, 2000, p. 88)

Although very young children learn to talk through joining in performances of conversations that are co-created and improvised between the child and more experienced others, as they grow older much of what children learn becomes routinized and rigidified into behavior (Holzman, 2000). An important distinction that Holzman makes here is between behaviour and activity: the former being a focus on the 'self-contained individual' and activity as what people engage in together 'rather than as the external manifestation of an individualised, internal process" (Holzman, 2000).

One activity that adults engage in which is clearly performative, in the sense of collectively creative, is improvisational drama, in which actors create scenes without a pre-determined script. I explore here ways in which problem solving could be considered similar to improvisational drama. Of course much of what passes for problem solving in school mathematics would be better described as exercises in that the method of solution is, in a sense, scripted and all the performer (child) has to do is replace certain elements. But problems for which pupils do not have a 'script' could, I argue, be understood as involving improvisation. Together, improvisation and emergence provide lens for examining problem solving activity and raise questions about assumptions and practices in teaching primary mathematics.

An example of collaborative emergent problem solving

The school context

This example comes from a two-year teaching experience in a primary school, Bow Bells, in the east end of London. The school is located in a traditionally working-class area but more recently there was also a high immigrant population, with many of the children starting school speaking almost no English. On measures of performance judged by national tests, only around 45% of pupils at age eleven were attaining the expected level 4 in the tests, compared with government targets of 80%. Inspection reports painted a picture of a school in difficulty, a consequence of which was that teachers were reluctant to apply to work there. The school was thus in a downward spiral. To counteract this, the local education authority had put in a new head-teacher, a specialist in literacy.

At that time I was looking to go back to do some school teaching. Several years of my own research had revealed little evidence of the sort of problem solving that was written about, and I had begun to wonder if teachers were right in sometimes thinking that academics in their ivory towers had got it wrong and that, given the constraints of schools, problem-solving based teaching was not possible. In approaching a local authority for a school to work in, Bow Bells was suggested.

At initial meetings with the teachers, two things were frequently commented on. First, teachers would talk about the limited language facility of the children (even for those children for whom English was their first language) and that consequently there was little point in asking the children to talk about mathematics. Second, and linked to the first point, there was a general sense that the children had little to contribute to mathematics lessons: it was important to equip children with the 'basics' before they would be able to engage in any form of problem solving. This attention to the 'basics' permeated throughout the school from the classes of five-year-olds to the eleven-year-olds and the predominant style of teaching across all the years was one of the teacher demonstrating a method on the board and the children subsequently completing practice worksheets.

The local authority was able to provide money for support in mathematics and so a colleague, Penny Latham, and I were able to work there more intensively than I originally anticipated: I was there one day a week for eight weeks each term and Penny there for two days a week, both of us over the course of two years. We agreed with the staff that our main focus would be on supporting the children in being able to talk about mathematics and to develop their mathematical understanding from problems and problem solving.

I have set out this context at some length as I want to make it clear that the children we were working with were not 'privileged' and did not have the kinds of teaching that might pre-dispose them to finding solutions to problems without being shown a method for doing this. The example that follows is typical of the sort of work we did. It comes from a class of six- and seven-year olds, one term into our second year of work with them and their teachers.

Improvised solutions: Jelly beans

The lesson started with a discussion about the idea of equal. I put up on the board

$$25 + 10 = 15 + 10 + 10$$

and asked the children to decide in their pairs whether they thought this was true or not. The class were all agreed that it was not true: because 25 + 10 made 35 not 15. Asked about the +10 + 10 that followed after the 15 and the children were clear that these were not relevant (one child suggested that the board had not been cleaned properly). Like many children of this age they had appropriated the idea that the equals sign means 'makes' and that what immediately followed it had to be the answer.

I talked about how the three numbers to the right could be added, representing this by adding them pairwise, 15 + 10 and 25 + 10, drawing lines down from the 15 and 10 and recording 25 below and then drawing a line down from the 25 and second 10 with the 35 below. Amid groans that I had (again) tried to 'trick' them, there was a general agreement that the statement was true. My recording stayed on the board for the rest of the lesson.

I then set up the main problem for the lesson, presenting it orally. I talked about visiting a friend, Richard, who ran a sweet shop and how he had posed a problem that he hoped the children could help with. He kept jars of different flavours of jellybeans from which he made up orders. I asked for suggestions as to the flavours of beans he might have, in the expectation that the children may have seen the Harry Potter movies and come up with some 'exotic' flavours. But they stuck with traditional fruit flavours, so I added in fish and broccoli. The six flavours were listed on the board and how many beans there were of each flavour:

Strawberry	72
Orange	23
Apple	33
Cherry	16
Fish	80
Broccoli	72

The problem was that Richard had an order for 300 beans and did not know if he had enough beans in total. Were the children able to find out?

There was a general murmuring of this being hard, but this was the second year of working with the children and they had come to accept that we would give them challenging problems to work on but also trust that they would get there in the end. In particular, good habits for working in pairs had been established, including that when working in pairs the children would share one piece of paper. They also knew that they could use any of the practical materials in the room and record their working in whatever way they found helpful.

A few pairs got out base-ten blocks to model the problem practically but most used paper and pencil. I want to examine the solutions of two pairs of children that are typical of the sorts of approaches the children took.

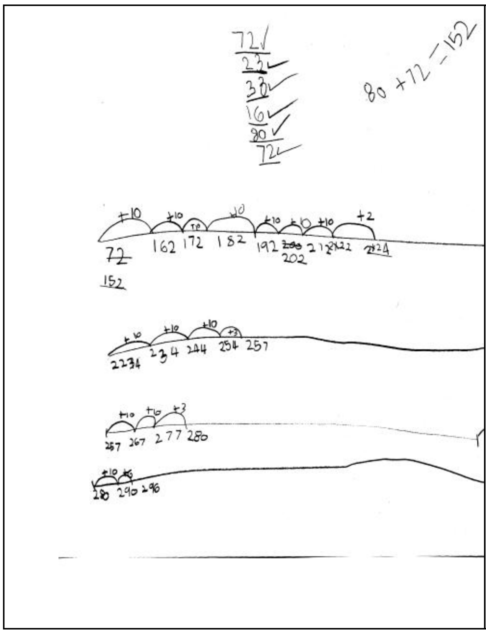

Figure 1. Amy and Ali's solution.

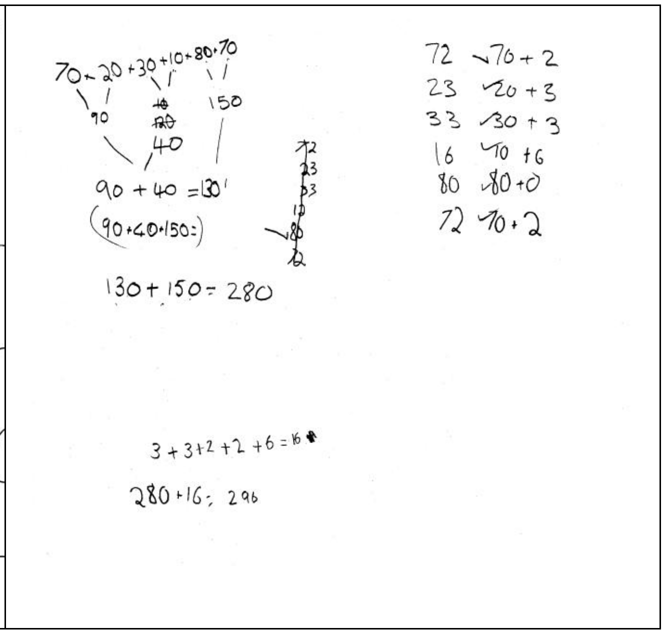

Figure 2. Ben and Beth's solution.

Sawyer (2000) argues for the analysis of collaborative emergence through examining group interactions, texts produced (including spoken texts), and the historical development of the group. As data were not systematically collected on the group interactions, I focus here on the physical texts that the pairs of children produced and then turn attention to the historical context that I consider supported the emergence of these texts.

Texts

Figure 1 shows the work of Amy and Ali. They copied down the numbers in the order in which they were on the board, but then started adding them systematically. They began with the largest pair, 80 and 72, adding these through the co-ordinated actions of Amy counting on in tens from 72 while Ali kept track of the number of tens added on. Both children put out fingers to keep track of action and keep their counting in time. Hence when Ali reached 80 Amy simultaneously reached 152. Then to add on the second 72 they turned to using the empty number line, drawing other number lines to add on 33, 23, and 16 in that order.

Figure 2 shows the work of Beth and Ben. Like Amy and Ali they started by copying down the list of numbers, ticking off 80 but then could not decide what to do next. Ben suggested writing the numbers down as tens and ones and they wrote the tens out, in the same order but horizontally, ticking them off as they went. It was not clear who chose to record the pairwise addition of the tens by appropriating the 'pull-down' notation that was on the board from the introductory activity.

Do these examples constitute examples of a collaborative emergent system?

Sawyer argues that a collaborative emergent system has the characteristics of:

> 1. unpredictability;
> 2. non-reducibility to models of participating agents;
> 3. processual intersubjectivity;
> 4. a communication system that can refer reflexively to itself, and within which the processes of communication themselves can be discussed; and
> 5. individual agency and creative potential on the part of individual agents
>
> (Sawyer, 1999, p. 453)

These solutions, and those of other children, were unpredictable given the range of solution methods. I had not planned to use the notation that morning that Beth and Ben used, but in the language of improv drama, it proved to be a ‘good offering’. In improv drama scenes, good offerings are ‘lines’ that open up possibilities for other players, as opposed to bad offerings that close things down. For example, in response to a simple opening of ‘Hi Mike’ ‘Hey sis’ would be a good offering, while ‘Who are you? I’ve never seen you before’ is a bad offering.

We knew the children well enough to know that the difficulty of the problem meant that no-one in class would have been able to solve the problem alone and the origins of the solutions cannot be reduced to an account of the understandings of individual children. The whole lesson was based on the processes of intersubjectivity together with individual agency and creativity. Sawyer’s fourth point is the least obvious, although the lesson finished with these pairs of children presenting their solutions to the class and a discussion of the clarity of each solution and which the children preferred.

Do these examples count as improvised?

It is easier to determine what is not improvisation than what is (Sawyer, 2000). Although we had worked with the children on using empty number lines, we had not used them for successive calculations as the children did here, and the use of the ‘pull-down’ notation was certainly improvised as the children had never been exposed to this before. Similarly we had never taught or observed the co-ordinated counting in tens activity of Amy and Ali. While the popular impression of improvisation is that it all has to be made up, it is more a sense of coordinating previously known and used elements in new ways, and it is in that sense I argue these are improvised solutions.

> Improvisation, like composition, is the product of everything heard in past experience, plus the originality of the moment. The contents of even a very accomplished improviser’s solos are not all fresh and original, but are a collection of clichés established patterns, and products of memory, rearranged in new sequences, along with *a few* new ideas. (Coker, 1964, p. 36, original emphasis).

Historical context

One aspect of the historical context is the attention to artefacts and tools that the children drew on. They were familiar with base-ten blocks. We had worked on fluency in adding multiples of ten, and emphasised the strategy of starting with the larger number when adding two numbers. We had introduced the children to the empty number line and had worked with it long enough for this to be a model for addition for many of the children (Gravemeijer, 1999).

But in addition to these 'cognitive' supports I want to make links to play and performance and the history of this that we, the class, had established, as I consider these as central to the children 'buying into' a problem that had a level of challenge beyond anything they had met before.

Becker (2000) in his analysis of jazz improvisation argues for the importance of having "a real shared interest in getting the job done" (p. 175). Like other researches leading to rich pupil solutions (for example, Fosnot & Dolk, 2001) the considerable time spent at the beginning of the lesson setting the context for the problem was not simply window dressing or a device to make some unpalatable calculations acceptable. There was a general 'suspension of disbelief' created by spending time setting up the scenario and in getting 'buy-in' from the children. This is not mere speculation: in the early part of setting up the problem, one of the girls repeatedly whispered to her neighbour "It's not true you know. He doesn't really have a friend with a shop." This increasingly became a stage whisper obviously intended to be overheard by everyone, so I stopped and we spent some time talking about whether it mattered if the 'story' was real or not. Although some of the children were disappointed that I would not reveal the truth, they were generally content to 'play along'. Such 'playing along' helps, I suggest, in the children being willing to 'play' with a problem. This is in contrast to some views that 'artificial' problems do more harm than good. While I would agree that the 'quick' word problem about shopping, followed by another about 'cooking' does not encourage engagement, I think more use could be made of more extended narrative scenarios to hook children in.

Discussion

Persons in environment

In her interpretation of the work of Vygotsky as a performative psychology, Holzman (2000) argues for being clear about *distinguishing* learners from their context but not treating them as *separate* from the context.

> While we surely can be (and are, in Western cultures) distinguished from environment, this does not mean we are separate from it. Instead of two separate entities … there is but one, the unity "persons-environment." In this unity, the relationship between persons and environment is complex and dialectical: environment "determines" us and yet we can change it completely (changing ourselves in the process, since the "it"—the unity "persons-environment"— includes us, the changers). (p. 86–87)

This has echoes of the emergence concept of downward causation (Campbell, 1974). "In downward causation, an emergent higher level property begins to cause effects in the lower level, either in the agents or in their patterns of interaction" (Sawyer, 1999, p. 455). Is it meaningful (or helpful) to talk of downward causation in the sense of the solutions that the children produce having some quasi-autonomous effect on the learners? In other words, is there a sense in which the solutions are manifesting themselves through the children, rather than the children are simply producing the solutions?

Experienced improvisers testify to downward causation. Although at the beginning of a scene, improvising actors have a whole range of options open to them (indeed, one of the disciplines of improv is to keep these options open for as long as possible), once the form and content of the scene starts to emerge, actors will talk afterwards of the scene 'writing itself'. Similarly jazz musicians report a sense of the music playing the band:

> The players thus develop a collective direction that characteristically—as though the participants had all read Emile Durkheim—feels larger than any of them, as though it had a life of its own. It feels as though, instead of them playing the music, the music, Zen-like, is playing them. (Becker, 2000, p. 172)

Even if it is only metaphorical to talk of downward causation, engendering a sense of this plays, I suggest, an important role in moving from either a teacher-centred or a pupil-centred lesson to a mathematics-centred one. The (tacit) sense of the solutions having some agency rather than being the 'property' of specific children may account for why that even at this young age the children were able to talk about the solutions without being defensive or possessive of them. Again there are resonances with jazz.

> Likewise, people must have a real shared interest in getting the job done, an interest powerful enough to overcome divisive selfish interests. In an improvising musical or theatrical group, for instance, no one must be interested in making a reputation or protecting one already made. (Becker, 2000, p. 175)

Conclusion

If it is the case that paired or group work that allows for collaborative emergence can result in more sophisticated, improvised, mathematical performance than could be achieved by the individual pupils then this has implications for the planning and implementation of lessons. First, most teachers base their planning on what they consider to be the needs of the individuals in their class which, as indicated here, are necessarily at a lower level of mathematics than could be achieved collectively. In the case of the children at Bow Bells school there was a clear pay-off from working at this more challenging level. These children were in Year 2, one of the years of primary schooling where the children have to take one of the externally set 'National Tests'. Not only did over 90% of the children reach the expected level on these tests (substantially higher than in previous years), but the children themselves commented on how easy they had found the test. Second a focus on the collective outcome presents challenges to the discourse of the individual that currently structures assessment activity.

References

Becker, H. S. (2000). The etiquette of improvisation. *Mind, Culture, and Activity 7*(3), 171–176.

Campbell, D. T. (1974). "Downward causation" in hierarchically organized biological systems. In F. J. Ayala & T. Dobzhansky (Eds.), *Studies in the philosophy of biology: Reduction and related problems* (pp. 179–186). Berkeley, CA: University of California Press.

Clark, A. (1997). *Being there.* Cambridge, Massachusetts, The MIT Press.

Coker, J. (1964). *Improvising jazz.* Englewood Cliffs, NJ: Prentice Hall.

Davis, B., & E. Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education 34*(2), 137–167.

Dweck, C. S. (2000). *Self-theories: Their role in motivation, personality, and development.* Philadelphia: Psychology Press (Taylor and Francis Group).

Fosnot, C. T., & M. Dolk (2001). *Young mathematicians at work: Constructing multiplication and division*. Portsmouth, NH: Heinemann.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning 1*(2), 155–177.

Holzman, L. (2000). Performative psychology: An untapped resource for educators. *Education and Child Psychology 17*(3), 86–101.

Sawyer, R. K. (1999). The emergence of creativity. *Philosophical Psychology 12*(4), 447–469.

Sawyer, R. K. (2000). Improvisational cultures: Collaborative emergence and creativity in improvisation. *Mind, Culture, and Activity 7*(3), 180–185.

Sawyer, R. K. (2001). Emergence in sociology: Contemporary philosophy of mind and some implications for sociological theory. *The American Journal of Sociology, 107*(3), 551–585.